Magnetic barriers in graphene

Reinhold Egger
Institut für Theoretische Physik
Universität Düsseldorf
A. De Martino, L. Dell’Anna
DFG SFB Transregio 12
Overview

Ref.: De Martino, Dell'Anna & Egger, PRL 98, 066802 (2007)

- Introduction to graphene
- Dirac-Weyl equation
  - Effects of disorder and interactions
  - Klein paradoxon
  - Inhomogeneous magnetic fields
  - (integer) Quantum Hall Effect
- Magnetic barrier
- Magnetic quantum dot

not discussed in this talk: superconductivity in graphene, bi- or multilayer, phonon effects etc.
Graphene

review article: Geim & Novoselov, Nat. Mat. 6, 183 (2007)

- Graphene monolayers: prepared by mechanical exfoliation in 2004 & by epitaxial growth in 2005 (but different properties!)
  
  Novoselov et al., Science 2004, Nature 2005, 

- „Parent system“ of many carbon-based materials (nanotubes, fullerene, graphite)

- Tremendous research activity at present
Graphene

- Monolayer graphene sheets (linear dimension of order 1 $mm$) have been fabricated
  - on top of non-crystalline substrates
  - suspended membrane
  - in liquid suspension
- Technologically interesting: high mobility (comparable to good Si MOSFET), even at room temperature
Graphene: a new 2DEG

- 2DEG represents surface state: possibility to probe by STM/AFM/STS techniques
- Electron-phonon coupling: spontaneous „crumpling“ of suspended monolayer reflects instability of 2D membrane  
  \[\text{Meyer et al., Nature 2007}\]
- Electronic transport
  - „Half-integer“ Quantum Hall effect
  - „Universal conductivity“ (undoped limit)
  - Perfect (Klein) tunneling through barriers
  - Aspects related to Dirac fermion physics
Graphene: Tight binding description

Basis contains two atoms; nearest-neighbor hopping connects different sublattices.

\[ a = \sqrt{3}d, \quad d = 0.14 nm \]

Wallace, Phys. Rev. 1947
Band structure

Exactly two independent corner points K, K′ in first Brillouin zone.
Band structure: valence and conduction bands touch at corner points \((E=0)\), these are the Fermi points in undoped graphene.

- Low energies: Dirac light cone dispersion
- Deviations at higher energies: trigonal warping

\[
E(\vec{q}) = \pm \hbar v |\vec{q}|
\]
\[
\vec{q} = \vec{k} - \vec{K}
\]
\[
v \approx 10^6 \text{ m/sec}
\]
Dirac Weyl Hamiltonian

Low energy continuum limit: massless relativistic quasiparticles

\[ H = H_K + H_{K'} = v \int d^2r \ \Psi^+ ( -i \hbar \nabla \cdot \vec{\sigma} ) \Psi \]

8 component spinor quantum field:
spin, sublattice, K point ("valley") degeneracy

\[ \Psi(x, y) = (\Psi_{K, \uparrow, A}, \Psi_{K, \uparrow, B}, \cdots, \Psi_{K', \downarrow, B}) \]

Pauli matrices in sublattice space: \[ \vec{\sigma} = (\sigma_x, \sigma_y) \]
Electron-electron interactions

- Kinetic and Coulomb energy both scale linearly in density $r_s$, interaction parameter $r_s$ not tunable by gate voltage
- Simple estimate: $r_s \approx 1$
  - RG theory: interactions scale to weak coupling
  - Fermi liquid theory holds, but not RPA
    
    Mishchenko, PRL 2007
  - Experiments observe near cancellation of exchange and correlation energy
    
    Martin et al., cond-mat/0705.2180

- no spectacular deviations from noninteracting predictions expected
  - Exceptions exist, e.g., asymmetric-in-$B$ part of IV curve
    
    De Martino, Egger & Tsvelik, PRL 2006

- In the following: disregard electron-electron interaction
Disorder effects

Two experimental puzzles
- Universal minimum conductivity $\sim 4e^2/h$
- Linear dependence of conductivity on doping

Novoselov et al., Nature 2005
Theoretical implications

Experimental data can be rationalized only if short-range impurity scattering suppressed

- Dominant mechanism: long-ranged Coulomb scattering by defects  
  *Nomura & MacDonald, PRL 2007*

- Then no K-K´ mixing

- Otherwise: strong localization expected  *Altland, PRL 2006*

- Universal „minimum conductivity“ currently subject to considerable & hot theoretical debate

  *Badarzon, Twordzydlo, Brouwer & Beenakker, cond-mat/0705.0886,*

  *Ostrovsky, Gornyi & Mirlin, PRB 2006*
Universal minimum conductivity?

Subtle issue…

compare order of limits for the optical conductivity of clean system at low frequency

\[
\lim_{\omega \to 0} \sigma(\omega, \ell = \infty) = \frac{\pi}{8} \frac{4e^2}{h}
\]

\[
\lim_{\ell \to \infty} \sigma(\omega = 0, \ell) = \frac{1}{\pi} \frac{4e^2}{h}
\]

Ludwig et al., PRB 1994

Disorder would have to increase conductivity to explain experimental data…
Klein tunneling

- Dirac fermions can perfectly tunnel through high and wide barrier
  - Electron and hole encoded in same equation (spinor!): Charge-Conjugation Symmetry
- Graphene provides good opportunity to study this effect  
  
- But: Confinement by electrostatic fields (gates) is then difficult

O.Klein, Z. Phys. B 1929

Williams, Di Carlo & Marcus, cond-mat 0704.3487
Electrostatic confinement

- Smooth electrostatic potentials: K-K´ scattering suppressed
- Single K point theory: Klein tunneling most pronounced for normal incidence on barrier, other states may be reflected
  
  *Silvestrov & Efetov, PRL 2007*

- How to produce mesoscopic structures? (quantum point contacts, quantum wires, quantum dots etc.)
- Our proposal: use magnetic barriers
Inhomogeneous magnetic field

Perpendicular orbital magnetic field

\[ \vec{B} = B(x, y)\hat{e}_z = \nabla \times \vec{A} \]

- Simplest level: ignore Zeeman field (and e-e interaction) electron spin irrelevant
- Consider ballistic case (for simplicity)
  - Disorder mostly of long-range type, preserves valley degeneracy
    - Nomura & MacDonald, PRL 2006
- For smooth field variation (on scale \( a \)):
  - K and K’ states remain decoupled, focus on single K point theory

Now: „minimal substitution“

\[-i\hbar \nabla \rightarrow -i\hbar \nabla + e\vec{A}\]
Dirac-Weyl equation with magnetic field

\[
\left( -i\hbar \nabla + e\vec{A} \right) \cdot \vec{\sigma} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \varepsilon \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}
\]

equivalent to pair of decoupled Schrödinger-like equations:

\[
\left( \left( -i\hbar \nabla + e\vec{A} \right)^2 + e\sigma_z B_z - \varepsilon^2 \right) \Psi = 0
\]

- Energies come in plus-minus pairs (chiral Hamiltonian)
- Zeeman-like term in sublattice space
Homogeneous field

\[ B(x, y) = B_0 \]

Relativistic Landau levels, 4-fold degenerate

\[ E_n = \text{sgn}(n) \nu \sqrt{2eB_0|n|} \]

results in „half-integer“ QHE because of presence of zero-energy state

\[ \sigma_{xy} = \frac{4e^2}{h} \left(n + \frac{1}{2}\right) \]

Experimentally confirmed

Integer QHE in graphene: expt. data
Magnetic barrier: Model

Consider square barrier: 

$$B(x, y) = \begin{cases} B_0, & |x| < d \\ 0, & |x| > d \end{cases}$$

Good approximation for 

$$\lambda_F > \lambda_B > a$$

Convenient gauge:

$$\vec{A} = B_0 \vec{e}_y \cdot \begin{cases} -d, & x < -d \\ x, & |x| < d \\ d, & x > d \end{cases}$$

$y$ component of momentum conserved!
**Magnetic barrier: Solution**

... pair of decoupled 1D Schrödinger eqns  
(assume electron-like state $\varepsilon > 0$)  
\[
\left(-\partial_x^2 + V_{A/B}(x) - \varepsilon^2\right)\psi_{A/B}(x) = 0
\]

Effective potentials  
\[
V_{A/B}(x) = \pm eA_y(x) + \left(p_y + eA_y(x)\right)^2
\]

parametrize momentum by kinematic incidence angle  
\[
k_x = \varepsilon \cos \phi
\]

\[
k_y = \frac{p_y}{\hbar} = \varepsilon \sin \phi + edB_0
\]

Gauge invariant velocity:  
\[
\vec{v} = v \begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix}
\]
Incoming scattering state (from left)

Left of the barrier: \( \Psi_{\text{left}} = e^{ik_x x} \begin{pmatrix} 1 \\ e^{i\phi} \end{pmatrix} + re^{-ik_x x} \begin{pmatrix} 1 \\ -e^{-i\phi} \end{pmatrix} \)

Under the barrier:

\[ \Psi_{\text{barrier}} = \sum_{\pm} c_\pm \begin{pmatrix} D_{-1+(el_B)^2/2} \left( \pm \sqrt{2}(k_y l_B + x/l_B) \right) \\ \pm i \frac{\sqrt{2}}{\epsilon l_B} D_{(el_B)^2/2} \left( \mp \sqrt{2}(k_y l_B + x/l_B) \right) \end{pmatrix} \]

Right of the barrier: \( \Psi_{\text{right}} = t \sqrt{k_x/k'_x} \ e^{i k'_x x} \begin{pmatrix} 1 \\ e^{i\phi'} \end{pmatrix} \)

with emergence angle in \( k'_x = \epsilon \cos \phi' \)
Perfect reflection regime

- Transmission/reflection probability
  \[ T = |t|^2, \quad R = |r|^2 = 1 - T \]

- Relation between emergence and incidence angle from y-momentum conservation
  \[ \sin \phi' - \sin \phi = \frac{2d}{\mathcal{E}l_B^2} \]

- No solution, i.e. perfect reflection, for low energy and/or wide barrier
  \[ \mathcal{E}l_B < d / l_B \]
  opens up possibility of confining Dirac Weyl quasiparticles
Transmission probability

angular plot of transmission probability $T(\phi)$ (away from the perfect reflection regime)
Magnetic quantum dot

- Circularly symmetric magnetic field \( \vec{B} = B(r)\hat{e}_z \)
- Total angular momentum \( J = -i\partial_\theta + \frac{\sigma_z}{2} \) is conserved, good quantum number \( j = m \pm 1/2 \)
- gives Dirac-Weyl radial (1D) equations

\[
\begin{pmatrix}
\psi_A \\
\psi_B
\end{pmatrix} = \begin{pmatrix}
e^{im\theta} \phi_m(r) \\
e^{i(m+1)\theta} \chi_m(r)
\end{pmatrix}
\]

\[
\begin{align*}
\frac{d\phi_m}{dr} - \frac{m + \varphi(r)}{r} \phi_m &= i\varepsilon \chi_m \\
\frac{d\chi_m}{dr} + \frac{m + 1 + \varphi(r)}{r} \chi_m &= i\varepsilon \phi_m
\end{align*}
\]

Magnetic flux through disc of radius \( r \) in flux quanta

\[
\varphi(r) = e \int_0^r r' dr' B(r')
\]
Simple model for magnetic dot

Again simple step-type model:  \( B(r) = \begin{cases} 
0, & r < R \\
B_0, & r > R 
\end{cases} \)

Solution:
\[
\phi_m(r < R) = a_m J_m(\varepsilon r) \\
\phi_m(r > R) = a_m \xi^{\frac{1}{2}} e^{-\xi/2} \\
\times \Psi \left( 1 + \tilde{m} \Theta(\tilde{m}) - \frac{\varepsilon^2 l_B^2}{2}, 1 + |\tilde{m}|; \xi \right)
\]

missing flux through dot
(in flux quanta)
\[
\delta = \frac{R^2}{2l_B^2} \\
\tilde{m} = m - \delta
\]

Matching problem gives energy quantization condition!
\[
\xi = \frac{r^2}{2l_B^2}
\]
Magnetic dot eigenenergies

(above zero, but below first bulk Landau level)

Estimate:

\[ B_0 = 4T \Rightarrow l_B = 13 \text{nm}, \]
\[ \varepsilon l_B = 1 \Leftrightarrow E = 44 \text{meV} \]

Energy levels tunable via magnetic field
Conclusions

- Graphene as model 2DEG system made of relativistic Dirac fermions
- Klein tunneling: Dirac fermions cannot be easily trapped by electrostatic fields
- Magnetic fields (inhomogeneous) can confine Dirac fermions. Solution discussed for
  - Magnetic barrier (square barrier)
  - Magnetic dot (circular confinement)